Technical Note

Introduction to the Trigonometric Shooting Reconstruction Method

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Abstract: This article is an introduction to the use of trigonometry for the purpose of reconstructing shooting scenes. It was written to provide the crime scene investigator, without a background in mathematics, with a basic look at how mathematical reconstruction is performed. This method has been reliable and is complementary to other reconstruction methods.

Introduction

String reconstruction is an acceptable method for showing the suspected path of a bullet at a shooting scene [1]. This method allows the investigators, members of the court, and the jury a way to visualize possible locations of the suspect or the victim in a shooting case. Stringing a shooting scene also shows the approximate flight path of the bullet relative to the rest of the scene. This, for example, may allow everyone to better determine the reliability of witnesses. It is helpful to emphasize that the exact flight path is nearly impossible to determine but that a very close approximation of the point of origin can be reliably determined.

The reconstruction of a shooting scene by the string method can sometimes pose several problems for investigators. Weather

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conditions at an outdoor scene, such as gusting wind, can make stringing the scene impractical, if not impossible. Terrain with trees or bushes that partially block the trajectory path, or low spots that place the path of the string out of reach, are also considerations. The sagging of the string becomes an issue as the investigator extends the analysis away from the point of impact of the projectile. Also, photographs of the reconstruction that accurately portray the angles and their relationships to each other can sometimes be difficult to obtain. This is usually not discovered until after the scene has been released.

An investigator who understands basic trigonometry can overcome these issues by calculating the flight path of the projectile using trigonometry. Unlike the string method, the trigonometric method is not inherently prone to error by interference of physical conditions at the scene. It can be used in conjunction with string data and scale drawings to help support conclusions. The trigonometric method allows the investigator to more easily determine the location of the trajectory at any point along the flight path. The measured data is also much more easily included in computer-generated scale drawings with programs such as AutoCad. The use of such programs is recommended because the dimensioning tools included can illustrate and compare trajectory paths from several bullets to determine their relative distances from each other. This is impractical and at times nearly impossible to perform with the traditional string method.

There are some outside influences that affect the accuracy of the trigonometric reconstruction method. The accuracy of any reconstruction method, including the trigonometric method, using the shape and location of bullet holes to determine the flight path can be negatively affected by deflection of the bullet during flight. The shape of any bullet hole should be carefully evaluated for any sign that the bullet was tumbling. Well-formed bullet holes or strikes are the most reliable for calculating angles. The user should also remember that minute errors, even those caused by rounding during calculations, are magnified when examining a trajectory path over long distances. Other factors, such as "density of target medium, bullet yaw, terminal velocity ..." [2], affect the margin of error but are generally estimated to cause less than 5% variation [2].

Special tools are required, such as a protractor, angle finder, and a scientific calculator. A digital angle finder is also required because it measures angles relative to the horizontal plane in .1 degree increments, whereas a traditional angle finder estimates angles in .5 degree increments [3]. A laser level or a clinometer is also essential for measuring the slope of the ground in situations that involve uneven or sloping terrain. These tools are not costly and can be obtained from a variety of suppliers.

It is important for those interpreting the data to understand that this method is to be used just as string data is used. It will not necessarily pinpoint the trajectory path but is capable of providing a very close approximation. The following will provide a basic overview of trigonometry and how it applies to shooting reconstruction.

Trigonometric Reconstruction Method

A brief explanation of the properties of a right triangle (a triangle containing a 90° angle) and some mathematical formulas must be presented first to provide the reader with the foundation upon which this method rests. Consider this triangle (Figure 1):

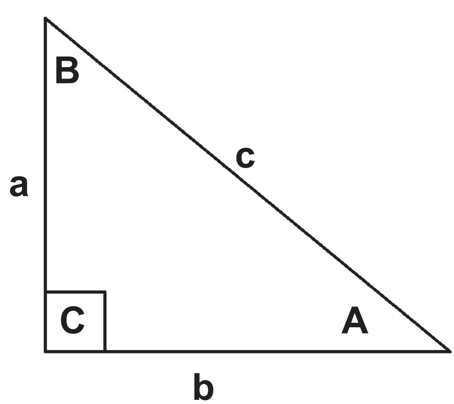

Figure 1

Right triangle.

Note that the triangle provided has three sides designated by lowercase letters and three angles designated by uppercase letters. You must first correctly label the triangle to be able to use the sine, cosine, and tangent functions. This means assigning the terms adjacent, opposite, and hypotenuse to the appropriate part of the triangle every time before calculations are performed. The hypotenuse of a right triangle is always the longest side so in this case *side **c*** is the hypotenuse. The opposite and adjacent sides are assigned relative to the angle that we are working with, so the adjacent side is always next to the angle in question and the opposite side is always across from the angle in question. For example, if you are considering *angle **A***, then *side **b*** becomes the adjacent side and *side **a*** is the opposite side. If you are using *angle **B*** for the calculation, then *side **a*** becomes the adjacent side and *side **b*** becomes the opposite side. This labeling is important because of the following rules of math:

1. $\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$

2. Sine (any angle) = $\dfrac{\text{opposite side}}{\text{hypotenuse}}$

3. Cosine (any angle) = $\dfrac{\text{adjacent side}}{\text{hypotenuse}}$

4. Tangent (any angle) = $\dfrac{\text{opposite side}}{\text{adjacent side}}$

You can now see that the sine, cosine, or tangent of any angle is equal to a ratio of two specific sides of the triangle. These rules allow us to calculate an unknown angle or side of any right triangle when we have any three other parts (including the known 90° angle that is always present in a right triangle).

There are two angles that are used for the trigonometric reconstruction method: (1) the horizontal impact angle (Figure 2), which is the right or left aspect of the impact, and (2) the angle from the flight path down to an imaginary horizontal plane that extends from the impact site (Figure 3). The latter angle represents the upward or downward aspect of the flight path.

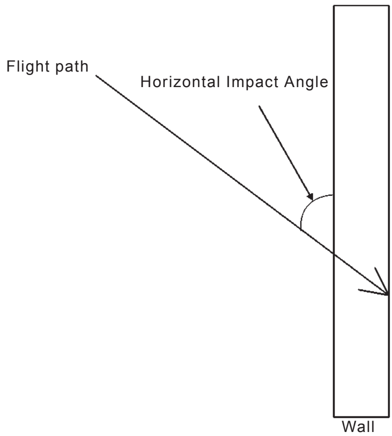

Figure 2

View from above.

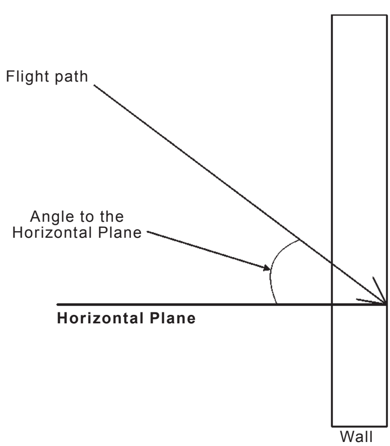

Figure 3

View from side.

To calculate the horizontal impact angle (marked as “x” in Figure 4), we imagine a small triangle inside of a wall (viewed from above) that a bullet has passed completely through. We can calculate one leg of the triangle by measuring the bullet hole on the front and back of the wall from a shared corner or edge (Figure 4). After obtaining these measurements, the side of the triangle is calculated by subtraction. The hypotenuse of this triangle is determined by passing a rod through the wall, marking the rod flush with the wall on both sides, and then removing the rod to obtain the measurement with a tape measure.

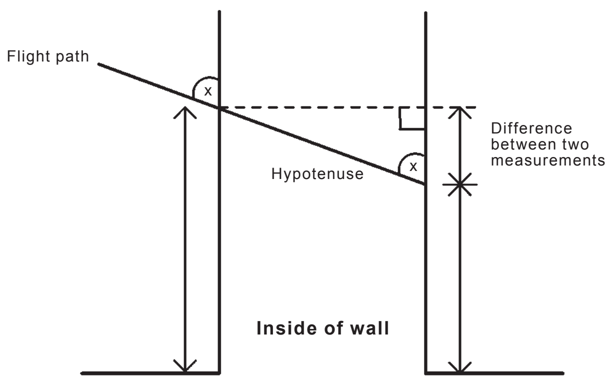

Figure 4

View from above.

Now we have the length of the adjacent side, with respect to angle x, and the hypotenuse. Using the cosine formula, we know the following:

$$\text{cosine (X)} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

We can change this formula algebraically to show that:

$$X = \cos^{-1}(\text{adjacent side/hypotenuse})$$

It is important while collecting this data to also indicate which direction, from the left or right, the flight path entered the wall.

Next, we will discuss the calculation of the angle to the horizontal plane. Once again, we must visualize a right triangle inside of the wall (viewed from the side) using the flight path for the hypotenuse (Figure 5). This will be the same hypotenuse that we measured by placing the rod through the wall while calculating the horizontal impact angle. The side of the triangle is calculated by the difference between heights of the bullet holes (front and back) from the ground.

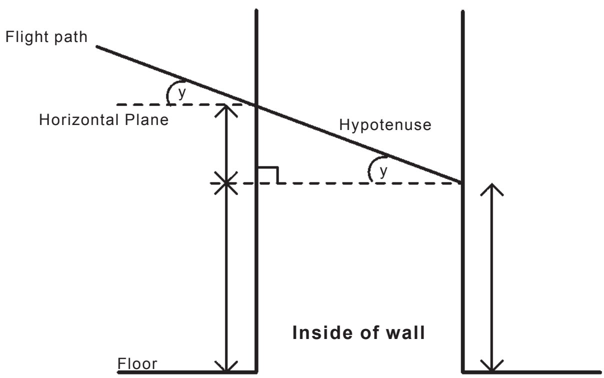

Figure 5

View from side.

We can use the formula for sine because we have the opposite side, with respect to angle y, and the hypotenuse as such:

$$\text{Sine (y)} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

We can change this formula algebraically to show that:

$$y = \text{sine}^{-1}(\text{opposite side/hypotenuse})$$

Now that we have the horizontal impact angle and the angle to the horizontal plane, the flight path of the projectile can be approximated. Extreme care must be taken when using this method to account for slope of the floor, particularly in outdoor scenes, and also in assessing the entry hole for irregularities that suggest the bullet was tumbling or had been deflected off of its true course.

Conclusion

Trigonometric reconstruction may be the only way to get accurate trajectory data in some scenes. It is also much easier to demonstrate, through the use of scale drawings, the actual distances between multiple trajectories. The authors have applied this method, as well as more advanced applications, in a variety of real shooting incidents. The data obtained has been very useful and is accurate when compared to the string data. This evidence is valuable to the reconstruction effort and is very useful for supporting the string method along with scale drawings.

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References

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